1 Theoretical Background of PELTON Turbine

1.1 Physics of Water Turbines (Impulse Turbines)

1.1.1 Basic Assumptions

The power output of an ideal turbine $P_T$ is derived from (a simplified) BERNOULLI’s equation with the following assumptions:

- no friction
- no viscosity
- incompressible medium
- laminar flow
- steady flow

The only parameters that have to be taken into account are the pressure and velocity differences at inlet and outlet of the turbine.\(^1\)

$$P_T = \left( \frac{p_1 - p_2}{\rho} + \frac{c_1^2 - c_2^2}{2} \right) \cdot \frac{dm}{dt}$$

where

$$\frac{p_1 - p_2}{\rho}$$

stands for the specific potential energy of the water flow, whereas

$$\frac{c_1^2 - c_2^2}{2}$$

denotes the specific kinetic energy. Thus the expression

$$P_T = \left( \frac{c_1^2 - c_2^2}{2} \right) \cdot \frac{dm}{dt}$$

describes the net power output of an ideal impulse\(^2\) turbine, where all potential energy is transformed into velocity (i.e. kinetic) energy.

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\(^1\) $c_i$ = linear flow velocity in \([m/s]\); $p_1$ = pressure in \([N/m^2]\); $\frac{dm}{dt}$ = mass flow in \([kg/s]\)

\(^2\) impulse turbines are e.g. PELTON, TURGO & BANKI-OSBERGER-MITCHELL crossflow turbines
1.1.2 Impulse Turbine

Impulse (i.e momentum) is defined as product of velocity and mass \( j = m \cdot v \), thus being a quantity of motion. The pulse force is defined as the first derivative of the momentum:

\[
\frac{d(m \cdot v)}{dt} = \frac{dj}{dt} = F
\]

In a closed system the sum of all acting forces equals zero, thus the forces of deflection, acceleration (positive or negative) are in equilibrium with the pulse force(s).

The runner of the turbine is driven by the negative acceleration of the peripheral flow of the jet, with flow velocity \( c_1 \). The product of torque \( M_T \) and the rotational speed of the turbine \( \omega \), which equals the power output \( P_T \), can be related to the flow rate \( Q = \frac{1}{\rho} \cdot \frac{dn}{dt} \) and the peripheral velocity \( u \) of the runner by means of:

\[
P_T = M_T \cdot \omega = \rho \cdot Q \cdot (c_1 - c_2) \cdot u
\]

with \( c_1 \) and \( c_2 \) the velocity components of the jet in the direction of \( u \) before and after touching the bucket.

The efficiency of the turbine \( \eta \) is given by the ratio of the power output at the turbine shaft \( P_T \) and the kinetic power \( P_{jet} \) of the water jet. This leads to a general equation for the turbine efficiency:

\[
\eta = \frac{P_T}{P_{jet}} = \frac{\rho \cdot Q \cdot (c_1 - c_2) \cdot u}{\frac{1}{2} \cdot \rho \cdot Q \cdot c_1^2}
\]

and then a variable \( k \)

\[
k = \frac{u}{c_1}
\]

is defined, which gives the ratio between the peripheral velocity of the runner \( u \) and the linear velocity of the jet \( c_1 \) (before hitting the runner). The optimal rotational speed \( \omega_{opt} \) can be calculated from the first derivative of the efficiency over \( k \):

\[
\frac{d\eta}{dk} = 0 (\omega = \omega_{opt})
\]

and the solutions for the PELTON turbine are given below.
Figure 2: PELTON turbine wheel plus nozzles
1.1.3 PELTON runner

If the vane of a PELTON turbine does not move, the only effect is to reverse the jet’s direction. Apart from some energy lost to friction (can be introduced by a “bucket efficiency” $\varphi_b$), the energy of the jet, and the magnitude of its velocity, stay the same as before.

- If the vane moves towards the jet, the water gains speed; if it pushes a vane moving away, it loses speed.
- In particular, if the water drives a vane moving at half its speed, then (neglecting friction) it loses all its velocity (and thus its kinetic energy) and just dribbles out of the moving vane.

The velocity of the jet is reduced by the friction losses in the nozzles – these losses are expressed by the friction loss coefficient $\varphi_n$ (0.96-0.99 for polished surfaces and well designed nozzle shapes), giving the jet velocity $c_0$ as

$$c_0 = \varphi_n \cdot \sqrt{2 \cdot g \cdot H} = \varphi_n \cdot \sqrt{\frac{2 \cdot p}{\rho}}$$

a function of head $H$ or pressure $p$ and it is generally assumed that $c_1 = c_0$ the inlet velocity at the runner equals the jet velocity at the nozzle (see below how to determine the nozzle efficiency).

It is possible to calculate the shaft power of the PELTON runner, assuming a deflection of the jet of $\theta = 165^\circ$ (this axial component of the reflected jet is necessary to prevent the deflected water from hitting the following buckets).

$$P_T = M_T \cdot \omega = F \cdot u = \rho \cdot Q \cdot (1 - \cos\theta) \cdot (c_1 - u) \cdot u$$

The efficiency of a PELTON turbine $\eta$ is then calculated from the following equation

$$\eta = \frac{P_T}{P_{jet}} = \frac{\rho \cdot Q \cdot (1 - \cos\theta) \cdot (c_1 - u) \cdot u}{\frac{1}{2} \cdot \rho \cdot Q \cdot c_1^2}$$

and this equation can be simplified by using the above given definition for $k$:

$$\eta = \frac{2 \cdot (1 - \cos\theta) \cdot (c_1 - u) \cdot u}{c_1^2}$$

$$\eta = 2 \cdot (1 - \cos\theta) \cdot (1 - k) \cdot k$$

showing us, how the efficiency $\eta$ depends on the ratio $k = u/c_1^3$.

As pointed out above, with the first derivative set to zero, we obtain the value of $k$ for maximum efficiency

$$k(\eta_{max}) = 0.5 = \frac{u}{c_1}$$

i.e. for maximum efficiency the runner tip speed $u$ should be equal half the jet velocity $c_1$. The maximum theoretical efficiency

$$\eta_{max} = \frac{1}{2} \cdot \varphi_n^2 \cdot (1 - \varphi_b \cdot \cos\beta_2)$$

is in the range of 0.95 for optimal values of the nozzle efficiency $\varphi_n$. Well designed PELTON reach an efficiency of 88-91% when they operate at 60-80% of full design flow.

1.2 Some Useful Relations for PELTON Turbines

In the following some useful relations are listed. Some are needed to evaluate the experimental results.

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3 Please see fig. 3 for a graph of this function
4 In real systems the optimal value for $k$ is between 0.45 and 0.49
Figure 3: Theoretical PELTON Runner Turbine Efficiency vs. k Value

Figure 4: PELTON Runner Relative Efficiency vs Rated Flow.
1.2.1 Pressure and Head

Pressure and Head in a hydropower plant are related by

\[ p = H \cdot g \cdot \rho \]  \hspace{1cm} (1)

or

\[ H = \frac{p}{g \cdot \rho} \]

with

\[ \rho_{H_2O} \approx 1000 \frac{kg}{m^3} \]

\[ g = 9.81 \frac{m}{s^2} \]

\[ p \left[ \frac{N}{m^2} \right] \]

\[ H \left[ m \right] \]

\[ [N] = \left[ \frac{kg \cdot m \cdot s^2}{s^2} \right] \]

1.2.2 Hydraulic Power

We assume the absence of friction losses

\[ P_{jet} = \frac{1}{2} \dot{m} c_1^2 \]  \hspace{1cm} (2)

kinetic power of jet flow equals hydraulic power of the water flow

\[ = p \cdot \dot{V} = p \cdot Q \]

and

\[ Q = \dot{V} = \frac{\dot{m}}{\rho} \]

thus

\[ P_H = H \cdot g \cdot \dot{m} \]  \hspace{1cm} (3)

the power \( P_H \) of the falling water can be expressed as

\[ P_H = Q \rho g H \]

1.2.3 How to calculate Turbine parameters

The speed of the turbine runner at pitch circle diameter (PCD) is given by

\[ u = \omega \cdot PCD/2 \]

with

\[ \omega = 2 \cdot \pi \cdot RPM/60 \]

Whereas the the speed of the jet \( c_1 \) has to be calculated by

\[ Q = \dot{V} = c_1 \cdot a_{jet} \]

\[ a_{jet} = \pi \cdot \frac{d_{jet}^2}{4} \]  \hspace{1cm} (4)

\[ c_1 = \frac{Q}{a_{jet}} = \frac{4 \cdot Q}{\pi \cdot d_{jet}^2} \]
Nozzle efficiency  If we divide the kinetic energy of the jet by the hydraulic energy from the pressure, we get an idea of the nozzle efficiency:

\[ \phi_{\text{nozzle}} = \frac{\frac{1}{2} \rho Q c_1^2}{pQ} = \frac{p c_1^2}{2p} \]

and \( c_1 \) already known.